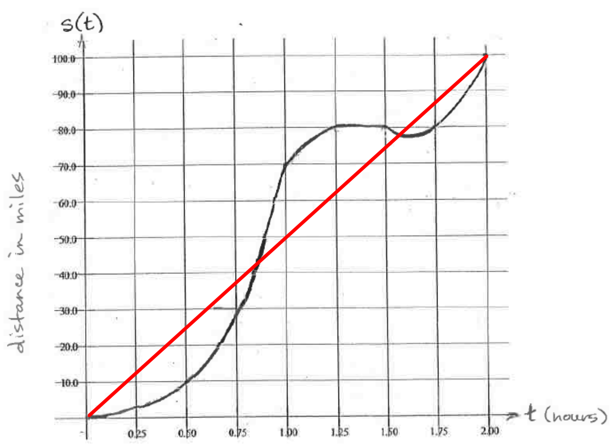
Calculus Warm Up #6-2

Compare tan worksheet in your group.

Applications of the Derivative
Practice #1

Name__________________________ Team_____

The graph below shows the distance (in miles) traveled by a car over the first two hours of a trip.



1. Find the average velocity for the entire trip.

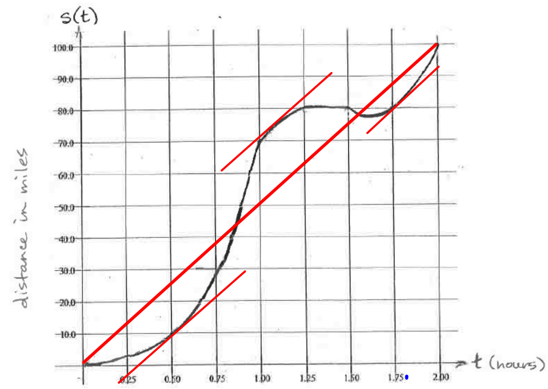
   slope of the secant

   50 mph

2. Find the average velocity in the second hour of the trip.

   30 mph

October 11, 2016



3. Is the car's velocity increasing or decreasing during the first half hour? How can you tell?

Tangent lines are getting steeper so slopes are increasing $\longrightarrow$ velocity is increasing.

4. Estimate when the instantaneous velocity was the same as the average velocity for the entire trip.

Mean Value Th. :)

Look for where tangents are parallel to secant line through endpts.

$t \approx 0.5,\ 1.1,\ 1.75$

5. When is the car traveling the fastest? Slowest?

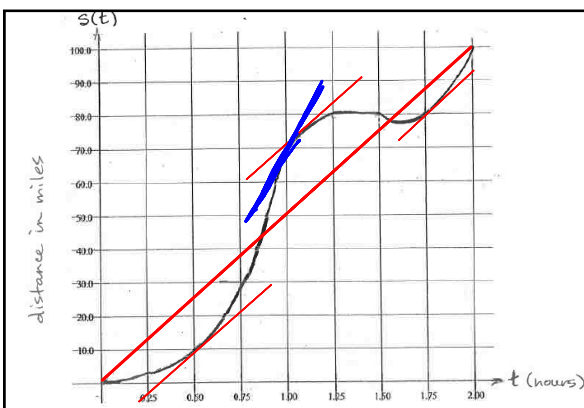
Speed $= |v|$

Fastest $\to$ steepest slope

$t \approx 0.875$ (PI)

Slowest where slope $= 0$

$t = 0,\ 1.6$ and on interval $(1.25, 1.5)$



4. Estimate when the instantaneous velocity was the same as the average velocity for the entire trip.

5. When is the car traveling the fastest? Slowest?

6. When is the car's velocity zero?

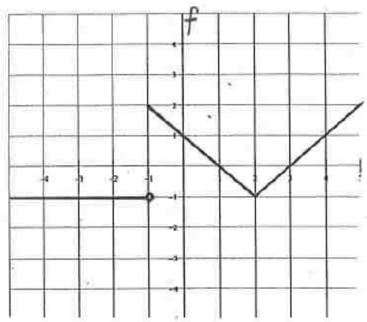
Same as

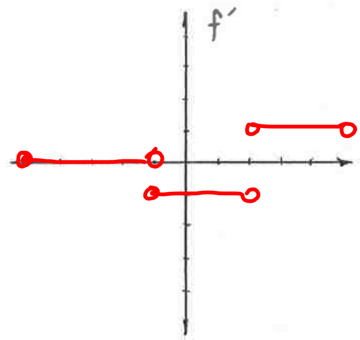
7. Is the instantaneous velocity when t = 1 hour greater or less than the average velocity for the entire trip?

Tangent @ $t = 1$ is steeper

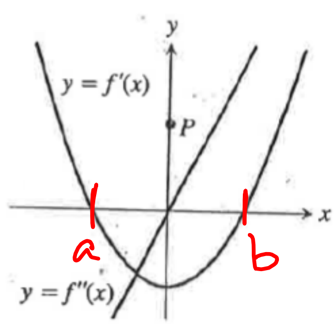
so velocity @ $t = 1$ $>$ Avg Vel. of trip

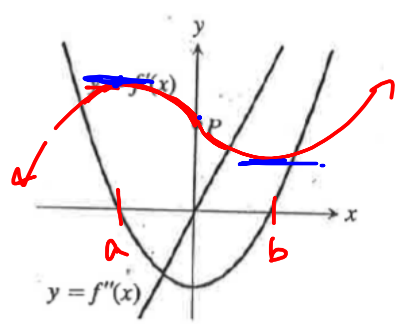
8. Given the function f, sketch f´





9. Given the graphs of the first and second derivatives of y = f(x) below, completely describe what you know about the graph of f, then sketch a possible graph of f to the right, that passes through point P.





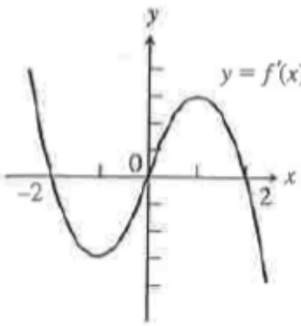
10. Given the graph of f ′, use the following to sketch a possible graph of f to the right.

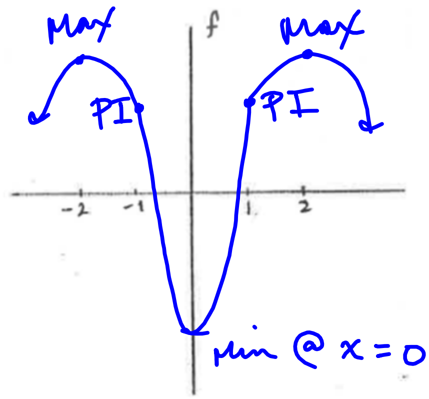
a) Where is f ′+/- ?

b) Where does f ′ change sign?

c) Where is f ′ increasing/decreasing?

d) Where will f be concave down/up?





p. 197 #1, 11

1. $y = x^3 - 3x^2 + 3$

11. $f(x) = x^4 - 4x^3 + 16x$

4.7

-Optimization problems

Many applications call for finding things like:

greatest profit
least cost
least time
greatest strength
optimum size
least distance

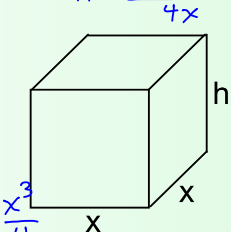
All are minimum or maximum values we can find with Calculus!

Ex: Design an open box with a square base and surface area = 108 sq. inches.

$108 = x^2 + 4xh$

$h = \dfrac{108 - x^2}{4x}$

What are the dimensions that would maximize the volume of the box?



$V = x^2h \rightarrow V = x^2\left(\dfrac{108 - x^2}{4x}\right)$

Plan to maximize volume: $V = 27x - \dfrac{x^3}{4}$
Find V', set = 0, $V' = 27 - \frac{3}{4}x^2$
get critical #'s,
confirm max.

$0 = 27 - \frac{3}{4}x^2$

$\frac{4}{3} \cdot \frac{3}{4}x^2 = 27 \cdot \frac{4}{3}$

$x^2 = 36$

critical # $\rightarrow$ $x = 6$

confirm max:

$V'' = -\frac{3}{2}x$

@ $x = 6$:

$= -\frac{3}{2}(6)$

$= -$ conc. dwn there

Confirms max!

Last step: Find h for dimensions

$h = \dfrac{108 - (6)^2}{4(6)}$

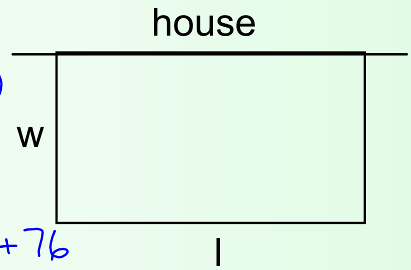
$h = 3$

Dimensions
6 in x 6 in x 3 in

Ex: For my garden, I want to enclose the largest rectangular area possible with 76m of fencing. I plan to use the house for one side of the fenced rectangle. What are the dimensions of the garden?

$\rightarrow 2w + l = 76$

$l = 76 - 2w$



$A = lw \rightarrow A = w(76-2w)$

$A = -2w^2 + 76w$

Plan to maximize area:
Find A´, set = 0,
get critical #'s,
confirm max.

$A' = -4w + 76$

$0 = -4w + 76$

critical # $\longrightarrow$ $w = 19$ $\longrightarrow$ confirm max:

$A'' = -4$

A″ is always negative
concave down everywhere!

Last find l & state dimensions:

$l = 76 - 2(19)$

$l = 38$

Dimensions: 38m x 19m

Steps:

1. Assign symbols for quantities given and to be determined, make a sketch if possible.

2. Write a primary equation for the quantity to be maximized or minimized. Consider its domain.

3. Reduce the primary equation to a single independent variable. (You might need a second relationship between the variables to do that.)

4. Take the derivative of the primary equation, find critical numbers, confirm extrema to answer the question.

HW:

p. 197 # 9, 19, 33

p. 203 # 5, 9, 11, 13